

## Confinement and scaling of the vortex vacuum of SU(2) lattice gauge theory\*

Kurt Langfeld, Hugo Reinhardt and Oliver Tennert

Institut für Theoretische Physik, Universität Tübingen  
D-72076 Tübingen, Germany

### Abstract

The magnetic vortices which arise in SU(2) lattice gauge theory in center projection are visualized for a given time slice. We establish that the number of vortices piercing a given 2-dimensional sheet is a renormalization group invariant and therefore physical quantity. We find that roughly 2 vortices pierce an area of  $1 \text{ fm}^2$ .

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## 1. Introduction

The conjecture [1] that confinement is realized as a dual Meissner effect by a condensate of magnetic monopoles recently received strong support by lattice calculations which show that in certain gauges the magnetic monopole configurations account for about 90% of the string tension [2, 3, 4]. The dominance of magnetic monopoles is most pronounced in the so-called Maximum Abelian gauge, where the influence of the charged components of the gauge field is minimized and which is a precursor of the Abelian projection, where the charged components of the gauge fields are neglected. The existence of magnetic monopoles is not restricted to the Maximum Abelian gauge. Monopoles occur, if the gauge fixing procedure leaves an  $U(1)$  degree of freedom unconstrained [5, 6]. The monopoles carry magnetic charge with respect to this residual  $U(1)$  gauge freedom.

If confinement is realized as a dual Meissner effect, i.e. by a condensation of the magnetic monopoles, the residual  $U(1)$  degree of freedom should be broken in the confining phase by the Higgs mechanism. This suggests that the relevant infrared degrees of freedom may be more easily identified in a gauge where the residual gauge is fixed. The  $U(1)$  gauge symmetry is explicitly broken in the Maximum Center gauge of the lattice theory, which has been implemented on top of the Maximum Abelian gauge [7]. The Maximum Center gauge preserves a residual  $Z_2$  gauge symmetry. Analogously to the Abelian projection, one has studied the center projection of  $SU(2)$  lattice gauge theory, i.e. the projection of  $SU(2)$  link variables (in Maximum Center gauge) onto  $Z_2$  center elements. The important finding in [7] (by numerical studies) has been a significant center dominance, i.e. the center projected links carry most of the information about the string tension of the full theory [7]. This naturally leads to the conjecture that the field configurations which are eliminated by the center projection are not relevant for confinement. Furthermore, the numerical calculations also reveal that the signal of the scaling behavior of the string tension is much clearer in center projection than in Abelian projection.

The center projection gives rise to vortices which are defined by a string of plaquettes. Plaquettes are part of the string, if the product of the corresponding center-projected links is  $(-1)$  (for details see [7]). The results in [7] indicate that these  $Z_2$ -vortices are the "confiners", i.e. the field configurations relevant for the infra-red behavior of the theory. This result supports a previous picture by 't Hooft [8] and Mack [9] in which the random fluctuations in the numbers of such vortices linked to a Wilson loop explain the area law.

In this letter, we will further investigate the properties of the vortices introduced by center projection on top of Abelian projection as considered by Del Debbio et al. in [7]. We will present a visualization of these vortices in coordinate space at a given time slice. The splitting of the vortices into branches is briefly addressed. By

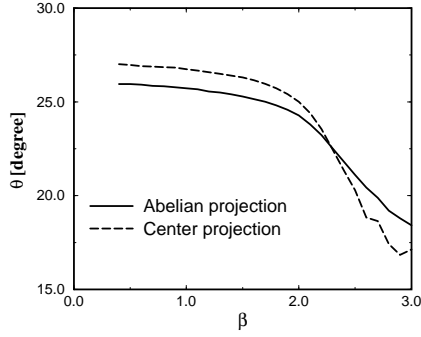


Figure 1: The angle  $\theta$  is a measure for the error induced by Abelian and center projection, respectively.

calculating the vortex distribution at several values of the inverse coupling  $\beta$ , we will establish that the string density is renormalization group invariant and therefore a physical quantity. On the average, we will find two vortices piercing an area of  $1 \text{ fm}^2$ .

## 2. The vortex vacuum in SU(2) lattice gauge theory

Center projection was defined in [7] on top of Abelian projection. The Maximum Abelian gauge [1] makes a link variable  $U_\mu(x)$  as diagonal as possible, and Abelian projection replaces a link variable

$$U_\mu(x) = \alpha_0(x) + i\vec{\alpha}(x) \cdot \vec{\tau}, \quad \alpha_0^2 + \vec{\alpha}^2 = 1 \quad (1)$$

by the Abelian link variable

$$A = \frac{\alpha_0(x) + i\alpha_3(x) \tau^3}{\sqrt{\alpha_0^2 + \alpha_3^2}} = \cos \theta(x) + i \sin \theta(x) \tau^3. \quad (2)$$

In order to quantify the error done by this projection, we introduce an angle  $\theta^A$  by

$$\tan \theta^A = \frac{\left\langle \sqrt{\alpha_1^2 + \alpha_2^2} \right\rangle}{\left\langle \sqrt{\alpha_0^2 + \alpha_3^2} \right\rangle}, \quad (3)$$

which measures the strength of the charged components relative to the strength of the neutral ones. The brackets in (3) indicate that the lattice average of the desired quantity is taken.

Center projection is then defined by assigning to each (Abelian) link variable a value  $\pm 1$  according the rule  $A(x) \rightarrow \text{sign}(\cos \theta(x))$ . The error which is induced by the center projection can be measured by  $\theta^C$ , i.e.

$$\cos \theta^C = \sqrt{\langle \cos^2 \theta \rangle}. \quad (4)$$

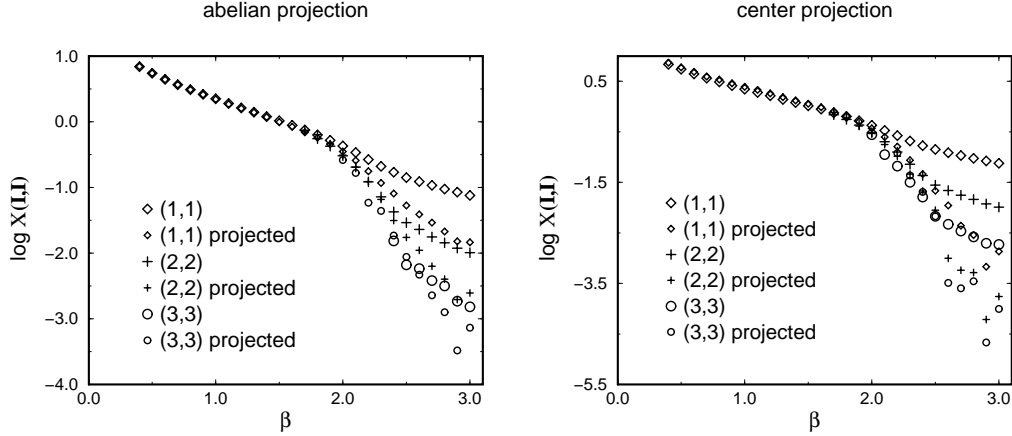


Figure 2: The Creutz ratios of the full SU(2) gauge theory compared with the values obtained in Abelian projected (left picture) and in center projected (right picture) theory.

Our numerical result for the angles  $\theta^A$  and  $\theta^C$  is shown in figure 1 as a function of the inverse coupling  $\beta$ . In the scaling window, i.e.  $\beta \in [2, 3]$ , the  $\theta$ -angle is of the order  $20^\circ$  degrees in either case. The relative error of a measured quantity on the lattice induced first by Abelian and subsequently by center projection is generically given by  $\sin \theta^A + \sin \theta^C \approx 75\%$ . Hence, the charged components of the link variable, i.e.  $a_1$  and  $a_2$ , are not small compared with the neutral components, i.e.  $a_0$  and  $a_3$ . In figure 2, we compare the numerical result for the Creutz ratios of full SU(2) lattice gauge theory with the result of Abelian projected and center projected theory, respectively. As observed by many people before [3, 7], the string tension is almost unaffected by the projection. There is no signal of an error of the order of 70% in the string tension as one would naively expect from figure 1. This shows that the degrees of freedom which are relevant for the string tension are largely untouched by the projection mechanism.

Center projection of lattice gauge theory induces magnetic vortices. We follow the definition of Del Debbio et al. [7]. A plaquette is defined to be part of the vortex, if the product of the center projected links which span the plaquette is  $-1$ . As explained in [7], a (center projected) field configuration contributes  $(-1)^n$  to the Wilson loop, where  $n$  is the number of vortices piercing the loop area. In particular, it was shown in [7] that the string tension vanishes, if the configurations with  $n > 0$  are discarded.

Stimulated by these results, we reduce the lattice ground state to a medium of vortices with the help of the center projection in order to construct a pure vortex vacuum. We then calculate the Creutz ratios for several values of  $\beta$  employing the vortex model. In particular, we measure the probability of finding  $n$  vortices which

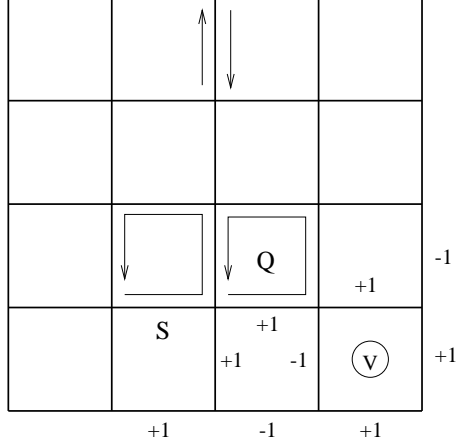


Figure 3: Schematic plot of the contributions of a center projected configuration on the lattice to the Wilson loop.

pierce the minimal area of the Wilson loop. The Wilson loop  $W$  is then obtained in the vortex vacuum model by

$$W = \sum_n (-1)^n P(n) / \sum_n P(n) . \quad (5)$$

The validity of this formula is easily checked: Define  $S_x \in \{\pm 1\}$  to be the center projected link (the index indicating the direction of the link is not shown) and  $Q_x$  the product of center projected links of a particular plaquette at position  $x$  (indices suppressed). If  $\mathcal{A}$  denotes the minimal area of Wilson loop and  $\partial\mathcal{A}$  its boundary, one finds (see figure 3)

$$\prod_{x \in \partial\mathcal{A}} S_x = \prod_{x \in \mathcal{A}} Q_x = (-1)^n , \quad (6)$$

where  $n$  is the number of vortices piercing the area. Here the first equality follows since center projected links  $S_x$  which do not belong to the boundary of the loop appear twice and hence give no contribution since  $S_x^2 = (\pm 1)^2 = 1$ .

The results for the Creutz ratios calculated from (5) as function of  $\beta$  is identical to the results obtained by taking the product of center projected links along the boundary of the Wilson loop (small symbols in the right picture of figure 2). Hence the vortex vacuum model reproduces almost the full string tension as well as the correct scaling behavior, if the continuum limit is approached.

### 3. Vortices scale

In order to get a more explicit picture of the vortex structure of the center projected gauge theory, we visualize the vortex distribution in coordinate space for a given time slice. In figure 4, we provide two generic configurations of such vortex filled state for  $\beta = 2.5$  and  $\beta = 3$ . The crucial observation is that the gas of vortices

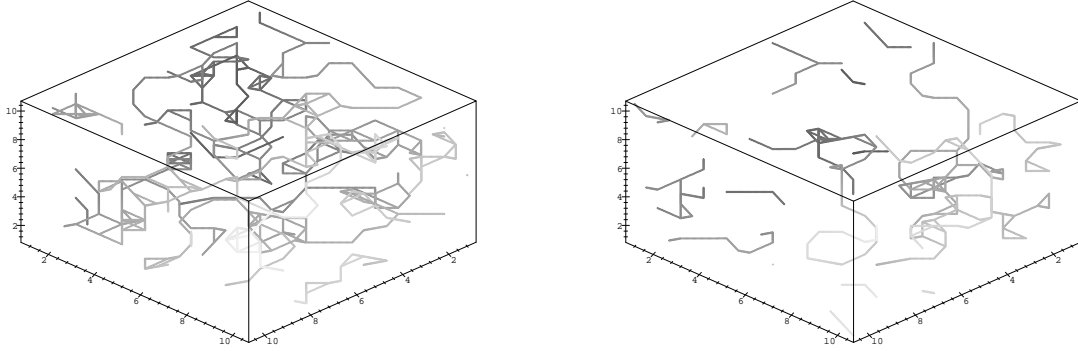


Figure 4: The distribution of the magnetic vortices in the  $10^3$  hypercube of the  $10^4$  lattice for a given time slice for  $\beta = 2.5$  (left) and  $\beta = 3$  (right).

is more dilute in the case of  $\beta = 3$  than in the case of  $\beta = 2.5$ . This behavior is anticipated, if we assume that the vortex gas is a physical object and therefore renormalization group invariant. From the Creutz ratios (see figure 2), one extracts the value  $\kappa a^2$ , where  $\kappa$  is the string tension (we use  $\kappa \approx (440 \text{ MeV})^2$ ) and  $a$  denotes the lattice spacing, as function of  $\beta$ . In practice, we extracted  $\kappa a^2$  from the Creutz ratios  $\chi(3, 3)$  using the center projected configurations, since there is a clear signal of the perturbative renormalization group flow in the center projected theory (see figure 2 and also [7]). From  $\kappa a^2$  the actual value of the lattice spacing  $a$  (in units of the string tension) is extracted for a given value of  $\beta$ . Larger values of  $\beta$  imply a smaller value of the lattice spacing due to asymptotic freedom. This implies that we zoom into the medium of the vortices, when we go to larger values of  $\beta$ , if the vortices are physical objects (like the string tension).

In order to establish the physical nature, i.e. the renormalization group invariance of the vortex medium, we investigate the average number  $N$  of vortices which pierce through an area of  $10^2 a^2$  as function of  $\beta$ . We then relate the lattice spacing  $a$  to the physical scale given by the string tension, and extract the density  $\rho$  of vortices which pierce an area of  $1 \text{ fm}^2$ . We find that  $\rho$  is almost independent of  $\beta$  in the scaling window  $\beta \in [2.1, 2.4]$ . This indicates that  $\rho$  is a renormalization group invariant and therefore physical quantity. Our results are summarized in table below.  $L$  denotes the spatial extension of our  $10^4$  lattice in one direction in physical units.

$\beta$	$N$	$\kappa a^2$	$L$ [fm]	$\rho$ [1/fm <sup>2</sup> ]	$\nu$
2.0	$22.0 \pm 0.3$	0.35	2.7	3.0	3.5
2.1	$18.9 \pm 0.4$	0.46	3.1	2.0	3.2
2.2	$15.1 \pm 0.6$	0.40	2.9	1.8	3.0
2.3	$11.1 \pm 0.4$	0.26	2.3	2.0	3.0
2.4	$8.0 \pm 0.7$	0.18	1.9	2.1	2.9
2.5	$5.8 \pm 0.9$	0.11	1.5	2.5	2.6

For  $\beta \leq 2$ , the lattice configurations are far off the continuum limit  $a \rightarrow 0$ , whereas the numerical uncertainties in  $\kappa a^2$  grow for  $\beta > 2.5$ . From the numerical results, we estimate

$$\rho = (1.9 \pm 0.2) \frac{1}{\text{fm}^2} . \quad (7)$$

Finally, we extract the number  $\nu$  of nearest neighbors which a particular point of the vortex has. It is a measure of the branching of the vortices. If the vortices form closed loops without branches, this number would exactly be two. If the vortices are open strings without branches, this number would slightly be smaller than two. The table above shows the numerical value of branching value  $\nu$  for several values of  $\beta$ . In the scaling window, a value  $\nu \approx 3$  is consistent with the data, while a significant deviation of  $\nu$  from 3 is observed for  $\beta \geq 2.5$ . This deviation is likely due to finite size effects.

#### 4. Conclusion

In this letter, we have studied the vortices arising in center projected lattice gauge theory, considered previously in ref. [7]. We have evaluated the Creutz ratios in a pure vortex vacuum defined via center projection, and have obtained almost the full string tension as well as the right scaling behavior towards the continuum limit. We have visualized the vortex distribution in coordinate space for a given time slice. In particular, we have obtained the density  $\rho$  of vortices piercing the minimal Wilson area as a function of  $\beta$ . We have observed an approximate scaling, which suggests that the vortices are not lattice artifacts but physical objects. On the average, we find  $\rho \approx (1.9 \pm 0.2)/\text{fm}^2$ .

Our investigations support the observations of ref. [7] that the vortices play the role of "confiners" in SU(2) Yang-Mills theory.

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